THE FUNDAMENTAL GEOMETRICAL APPROACH OF THE OBJECTS OF THE EARTH:

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Abstract:
What is an Object? It is an expression of nature in such a way that it can pursue the purpose of the expression in the most prominent way. Thus, nature designs the shapes of the objects of every matter in the most purposeful way to attend to its highest possible expression. Moreover, the process of gaining these maximum possibilities of expression is called evolution. However, in this process, nature follows a Particular format or geometrical Shape to express itself, which is the primary concern of the Paper.

Introduction:
Whatever we notice in nature, from sky-touching Mountains to small ferns or the colour patterns of a butterfly's wings, all are expressed and identified in a particular size and Shape. This size and Shape are the key functions that enable us to understand what the object is and its role in nature. However, the number of distinct scales of length of natural Patterns is, for all practical purposes, infinite, (B.Mandelbrot;1983). Is it always true? Alternatively, are there any particular fundamental patterns that can analyze the tendency of nature that exhibits a higher degree and a different level of complexity in forming shapes (B.Mandelbrot;1983). On this note, we can consider the judgment that some natural forms, like honeycombs, are symmetrical, and most fractals display a hidden logical order despite their apparent disorder. This underlying logic can be described mathematically, revealing that fractals emerge from repetitive algorithms, which lead to self-similar structures (P. Ball, 2016). However, in this Paper, the objects of nature will be fragmented into geometrical Shapes, and wish to derive the most fundamental geometrical Shape of nature.

Review of Literature:
Euclidean Geometry postulates that a Point has no parts, no magnitude, no size, and thus cannot be divided into parts, but points have positions (Euclid et al.), which is the foundation of all geometric Shapes. Again, the basis (extremities) of the line is the Point. At the same time, a line is length without breadth (E.Layng). A plane angle is the inclination of two lines to each other in a plane, which meet but are not in the same direction. Again, a plane rectilinear angle is the inclination of two straight lines to each other in a plane, which meet but are not in the same straight line (E.layng). By considering this, we can derive that when two different collections of points are inclined to each other but in different directions, they form angles, just as when two straight lines or the summation of points with different directions are inclined to each other form rectangular angles.

However, the Circle is a Surface plane figure contained by a Single line called the circumference, and is such that all straight lines drawn from a certain point within the figure to the circumference are equal. This Point is called the centre of the Circle (Euclid et al). Repeatedly, we can say that the circumference of a circle is a collection of Such points that continuously follow a Single point. Moreover, the distance between the centre point and the points forming the circumference is always the same. As the points have no magnitude but have position, when several positions with n directions surround the focus point, then a closed surface originates, called a Circle. Moreover, every Point of a circle has an outwardly directional tangent. This gives them a continuity with no beginning or ending Point. That continuity is most important for forming the Circle. If we cut the part of the circumference that is the arc of the Circle, the arc of the Circle is a line with two extreme points (Euclid et al.).

For the trilaterals, as they do not originate from any particular point. A trilateral figure is defined as a figure that is contained by three straight lines and enclosed (Euclid et al). However, there is no Mention of mandate continuity as no different collections of points are inclined at each other and enclose a specific surface plane inside the inclination. Any three distinct points on a circle form a unique triangle(Euclid et al).

Thus, in this literature, we shall analyze different Symmetrical shapes in nature and try to find out is there any fundamental Shapes or not who are the extremites of all other Shapes of Earth.

Objectives:
1. To study the fundamental basis of Geometrical shapes.
2. To study the geometrical Pattern in earthly Materials.
3. Try to findout the most fundamental Shapes of the earth.
4. To implement the hypothesis of the Paper in different 3d objects around us.

Hypothesis:
1. There is a unique, definite geometrical pattern in nature, but the degree of rearrangement is infinite. The arrangement of these fundamental shapes creates a different appearance for any object.
2. Two geometrical forms- i) the Circle, which strictly follows the centre of its origin, and ii) the Triangle, which follows the Circle but not the origin. These two Shapes are rearranged in all Symmetrical Examples of Nature.

Methodology:
Conceptualize the idea of Point, Circle, straight line, angle and Triangle by Euclidean and Indian philosophical schools of thought. Moreover, 11 Different Euclidean geometric Shapes are studied and fragmented into geometric Shapes. 11 Symmetrical objects of nature, Such as honeycombs, raindrops, and the Shape of mountain trees, are being cautiously picked to represent different patterns and analyse the pattern of occurrence. 11 Symmetrical artificial and natural 3dimensional objects are chosen and fragmented into geometric patterns.

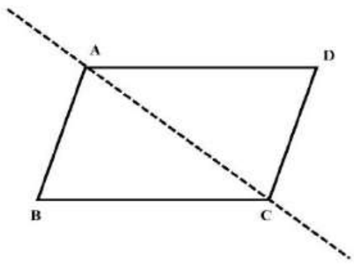
Tools used:
Basic Geometric Tools - pencil, compass, protector, vertices, Pencil, eraser, graph paper. Software: MS Word, Adobe Illustrator, Geometrize.
Result:

Exemplifying through Postulates of Euclid: In this chapter, we shall take the example from 11 different shapes and prove that the Triangle is their basis.

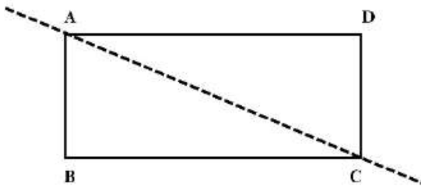
Euclidean Basis
1. **Book I, Proposition 35**: "Parallelograms which are on the same base and between the same parallels are equal in area."
2. **Book I, Proposition 41**: "If a diagonal bisects a parallelogram, the triangles on either side of the diagonal are equal in area."

So, Parallelograms are:

Type	**Sides**	**Angles**	**Diagonals**
Parallelogram	**Opposite equal**	**Opposite equal**	**Bisect each other**
Rectangle	**Opposite equal**	**All 90°**	**Equal**
Rhombus	**All equal**	**Opposite equal**	**Perpendicular**
Square	**All equal**	**All 90°**	**Equal & perpendicular**
Rhomboid	**Opposite equal**	**Not 90°**	**Bisect each other**



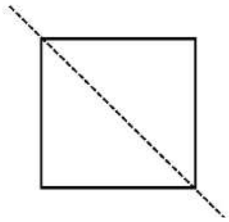
Parallelogram: for the parallelogram ABCD if we cut across its diagonal AC or BD, we shall get two triangles ΔABC and ΔACD.
According to the Euclide Postulates Summation of the area of these two triangles is equals to the area ■ABCD.
Area of ■ABCD = area of ΔABC + ΔACD



Rectangle: For the Rectangle ABCD if we cut across its diagonal AC or BD, we shall get two triangles ΔABC and ΔACD. According to the Euclide Postulates Summation of the area of these two triangles is equals to the area of the rectangle ■ABCD.

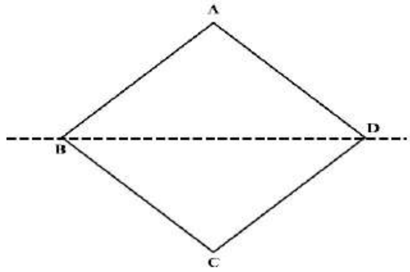
Area of ■ABCD = area of ΔABC + ΔACD

Squares: As all squares are rectangle (Euclid et al). thus, this formulation is also valid for the Squares also.



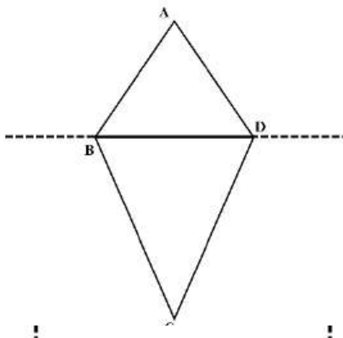
Rhombus:

For the Rhombus ABCD if we cut across its diagonal AC or BD, we shall get two triangles ΔABD and ΔBCD. According to the Euclide Postulates Summation of the area of these two triangles is equals to the area of the Rhombus ■ABCD.
Area of ■ABCD = area of ΔABC + ΔACD



Rhomboid:

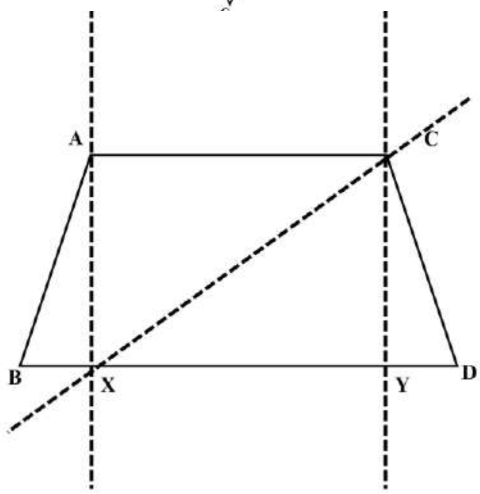
For the Rhomboid ABCD if we cut across its diagonal AC or BD, we shall get two triangles ΔABD and ΔBCD. According to the Euclide Postulates Summation of the area of these two triangles is equals to the area of the Rhomboid ■ABCD.
Area of ■ABCD = area of ΔABC + ΔACD



Trapezium:
For the Trapezium, also follow the Same Pattern. Where the area of tl

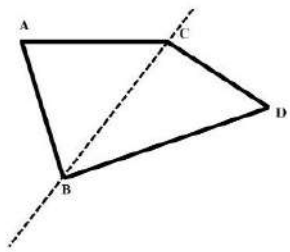
$$= \frac{1}{2}(a + b)h$$

a = length of one side of the trapezium
b = length of another side of the trapezium
h = height or distance between top and base



Moreover, it can be written as:

For a trapezium, also follow the Same Pattern. Where the area of the ABCD trapezium is: ½(a+b)h but it can be seen as:
ΔABX and ΔCYD are two triangles and AXCY is a rectangle and if we cut the rectangle across its diagonal two another triangles will be found which are ΔACX and ΔXYC.
Thus, we can say that: The area of the trapezium ABCD = area of ΔABX + ΔCYD + ΔXCY+ ΔACX



Irregular Quadrilateral:
If ABCD is an irregular Quadrilateral,
When the sides and one diagonal are known (by dividing into two triangles)

If you draw a diagonal (say AC) in quadrilateral ABCD, it splits the figure into two triangles: △ABC and △ACD.
Find each triangle's area using **Heron's formula** and then add them.
For △ABC:

$$s_1 = \frac{AB + BC + AC}{2}$$

$$A_1 = \sqrt{s_1(s_1 - AB)(s_1 - BC)(s_1 - AC)}$$

For △ACD:

$$s_2 = \frac{AC + CD + DA}{2}$$

$$A_1 = \sqrt{s_2(s_2 - AC)(s_2 - CD)(s_2 - DA)}$$

Then total area:

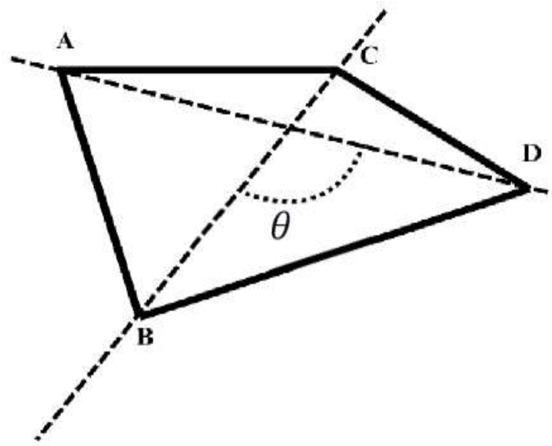
$$A = A_1 + A_2$$

Or,

When the lengths of the diagonals and the angle between them are known

Let the diagonals be d_1 and d_2, and the angle between them be θ. Then the **area (A)** is given by:

$$A = \frac{1}{2} d_1\, d_2 sin\theta$$

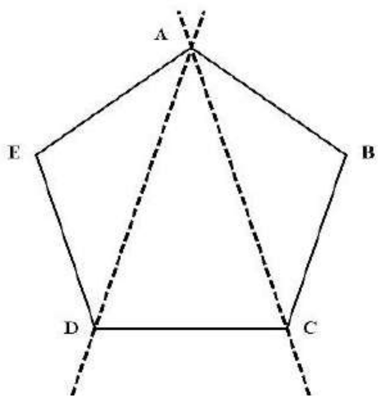


Hence, it can be Said that the extremes of the Euclidian quadrilaterals are Triangle.

Now we shall check for the Polygons:

Pentagon:

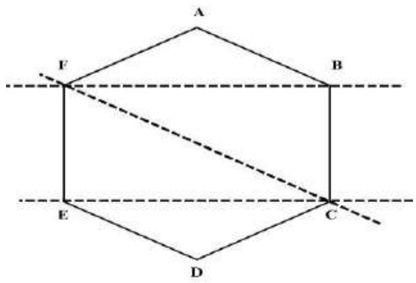
Here **ABCDE** is a Pentagon. Which is cut by joining the alternative points. Which is formed three triangles respectively which are ΔAED, ΔABC, ΔACD



∴**area of Pentagon ABCDE= area of** (ΔAED + ΔABC + ΔACD)

Hexagon:

Here **ABCDEF** is a Hexagon. Which is cut the alternative points. Which is formed triangles respectively which are
ΔAFB, ΔECD, ΔFEC, ΔFBC

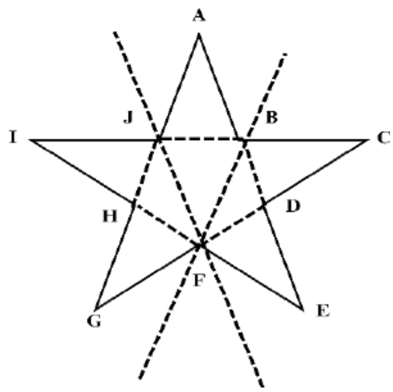


(As we Proved previously the rectangle is the summation of two triangles.)

∴**area of Hexagon ABCDEF= area of** (ΔAFB + ΔECD + ΔFBC+ ΔFEC)

The 5 side Star:

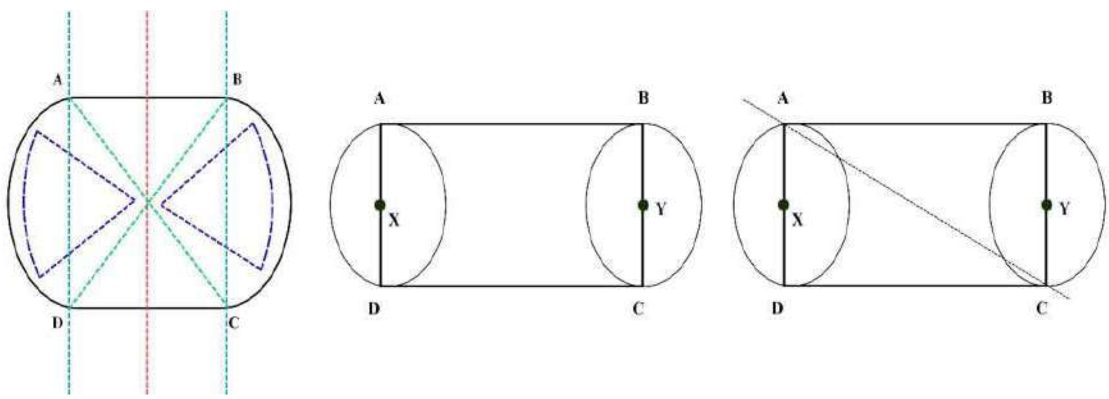
Here ABCDEFGHIJ is a five Side Star or a form of Decagon. Which is cut the alternative points. Which is primarily formed 5 triangles respectively which are ΔAJB, ΔBCD, ΔDEF, ΔFGH, ΔHIJ, and a pentagon JBDFH. It is already shown in this paper that the Pentagon is a collection of three triangles.



For pentagon JBDFH the triangles are ΔJBF, ΔJHF, ΔBDF

∴**area of Decagon ABCDEFGHIJ= area of** ($\Delta AJB + \Delta BCD$+ΔDEF + ΔFGH+ $\Delta HIJ + \Delta JBF$ + ΔJHF+ ΔBDF)

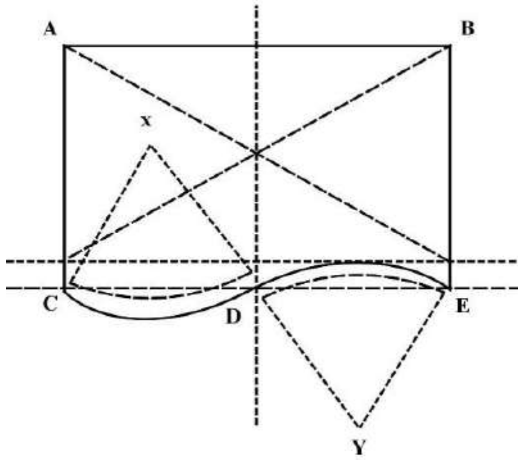
The Composed Shape of Circle and Rectangle: 1st observation:



Here ABCD is a rectangular Shape which is Composed of a rectangle ABCD and two eclipse whose center are X,Y respectively. Here the rectangle ABCD is the Summation of two triangle ΔADC, ΔABC (all parallelograms are summation of two triangles (Euclid et al)).

∴**area of rectangular ABCD** = $\frac{1}{2}$(area of parabola center X) + $\frac{1}{2}$(area of parabola center Y) + area triangle of ΔADC + area of triangle ΔABC

2nd observation:



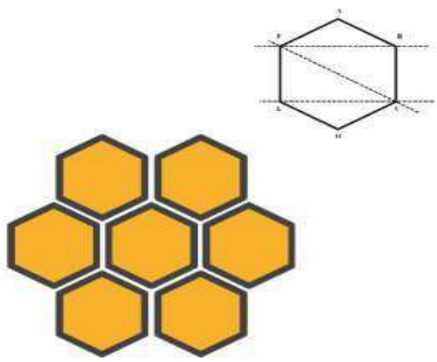
Here ABCE is a rectangular shape whose side CE is not a straight line rather addition of two arcs CD and DE. whose center are X and Y. they are situated inward and outward of the rectangle.
Thus, the area of the Shape ABCE is = Area(ΔABE)+Area of the curved region between C−D−E.

If the curves are circular arcs (centres and radii known)
Suppose the top bounding curve between $C$and $E$is formed by one or more circular arcs. The area of a circular segment of radius $r$that subtends central angle θ(in radians) is

$$\text{Area Segment} = \frac{r^2}{2}\,(\theta - \sin\theta)$$

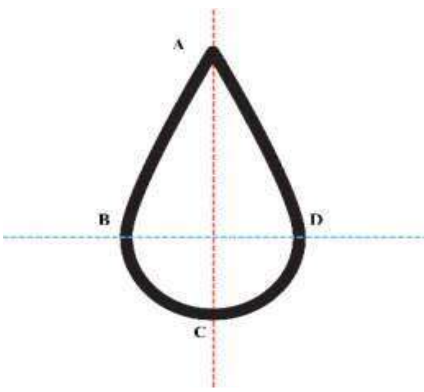
If curved region is the union/difference of several circular segments, sum the appropriate segment areas (adding if the segment is inside the region, subtracting if it is a hole).

In this Part a Documentation has been listed of different Geometrical Figures which are directly observed in nature without any change.



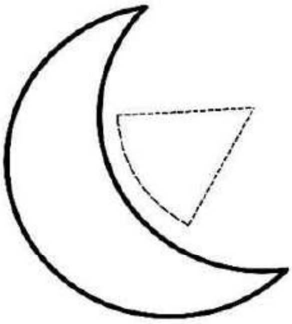
The Honeycomb and the Benzene Ring:

here is a Structure of benzene ring/ honey comb and they are hexagon in geometric figure. Previously we have been proved that the hexagon is collection of four nos. of triangle. Hence, it can be said that the extremities geometric shape of honey comb are triangles.



The water Droplets:
Here is a structure of Water droplets ABCD which is an assimilation form of Δ and arc BCD. Thus,

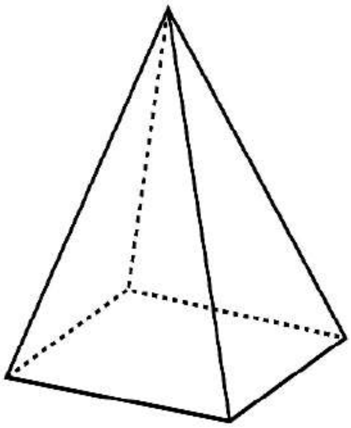
∴The area of ABCD = area of Δ ABD **+ area enclosed by the arc BCD**

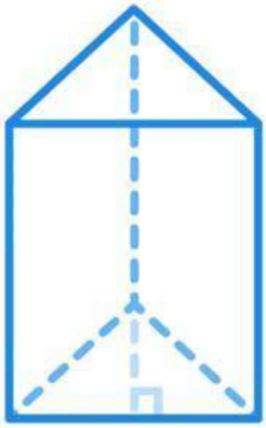


And as water droplets are extremes of any water bodies hence it can be said that, basic structure of the all of the water bodies are triangle and arc.

The Lunar Crescent:
The Lunar crescent is Such a structure which is formed by iclination of two arcs on one another, and the center of both arcs is towards the angular head of the cone.

The pyramid and Prism:

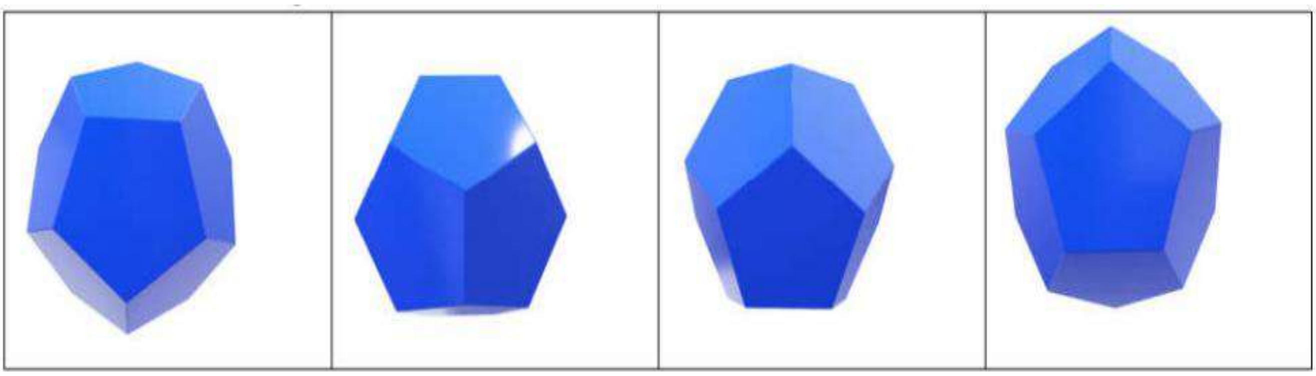


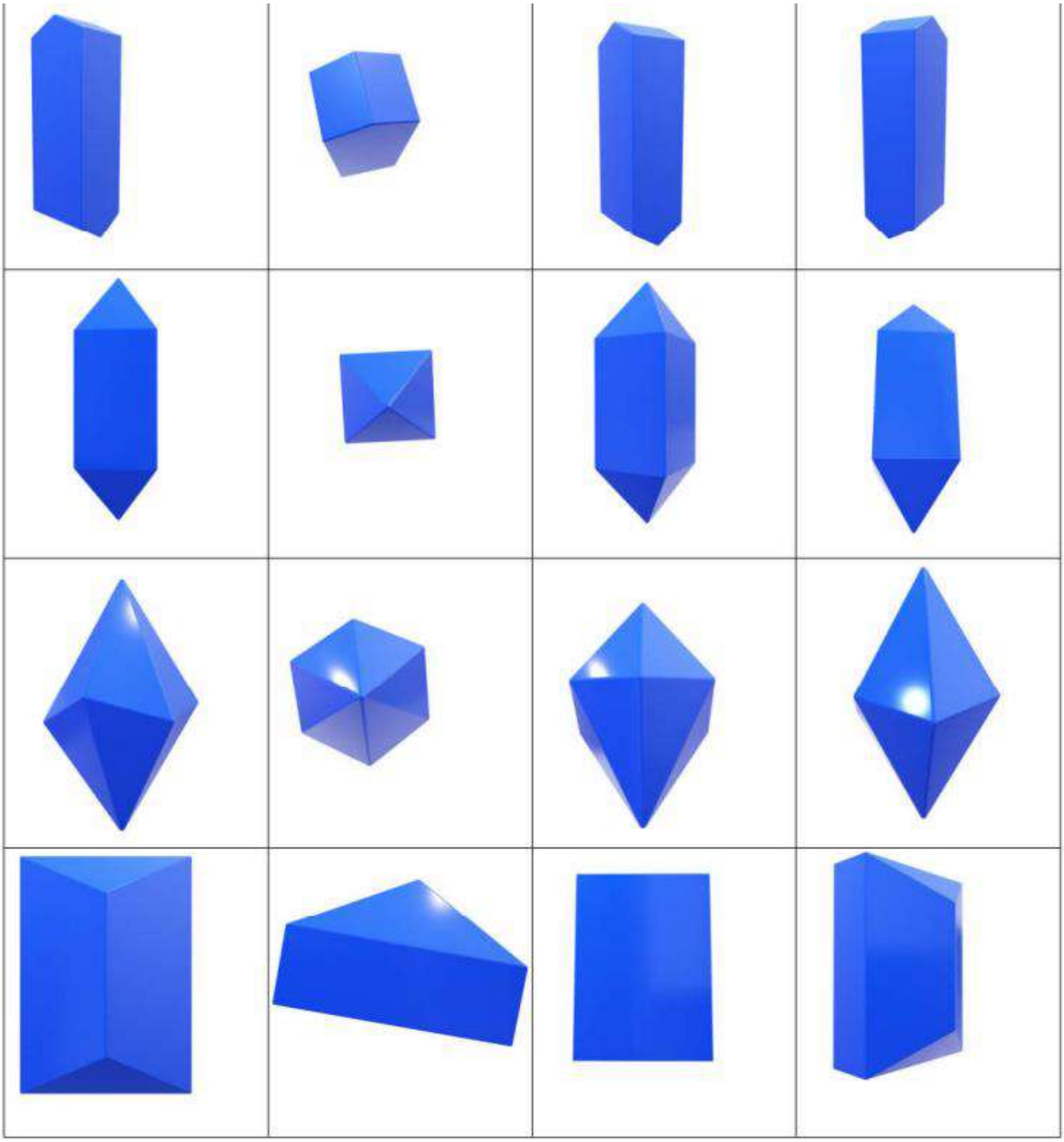


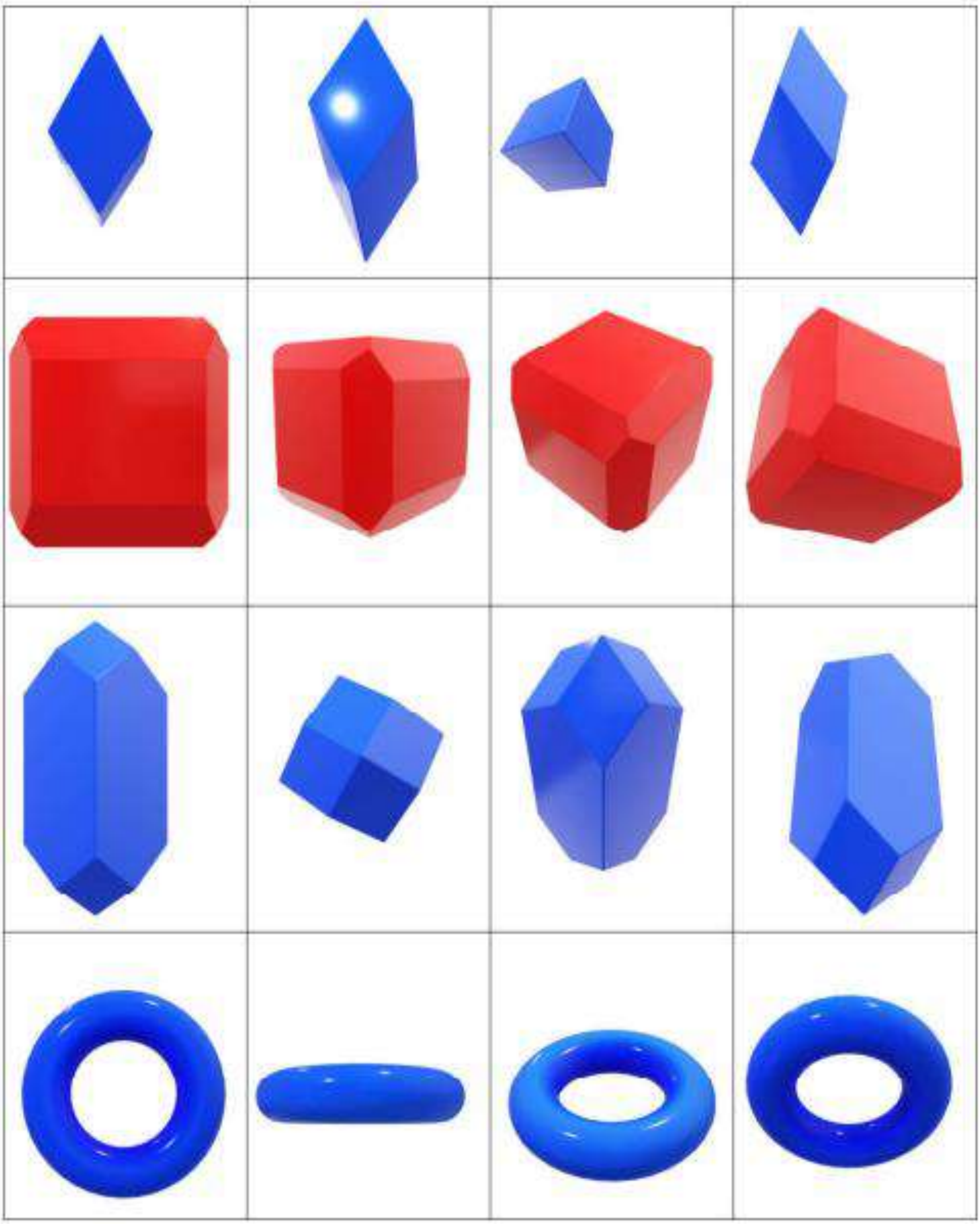
the Pyramid and Prism (a form of Pyramid) both are made with different arrangement of triangles and rectangles (Visibly). And as the arrangement alters the pattern of Pyramid also alters. In that note, Prism is also a form of Pyramid which consists of two triangle and three rectangle which can be separated as six individual triangles.

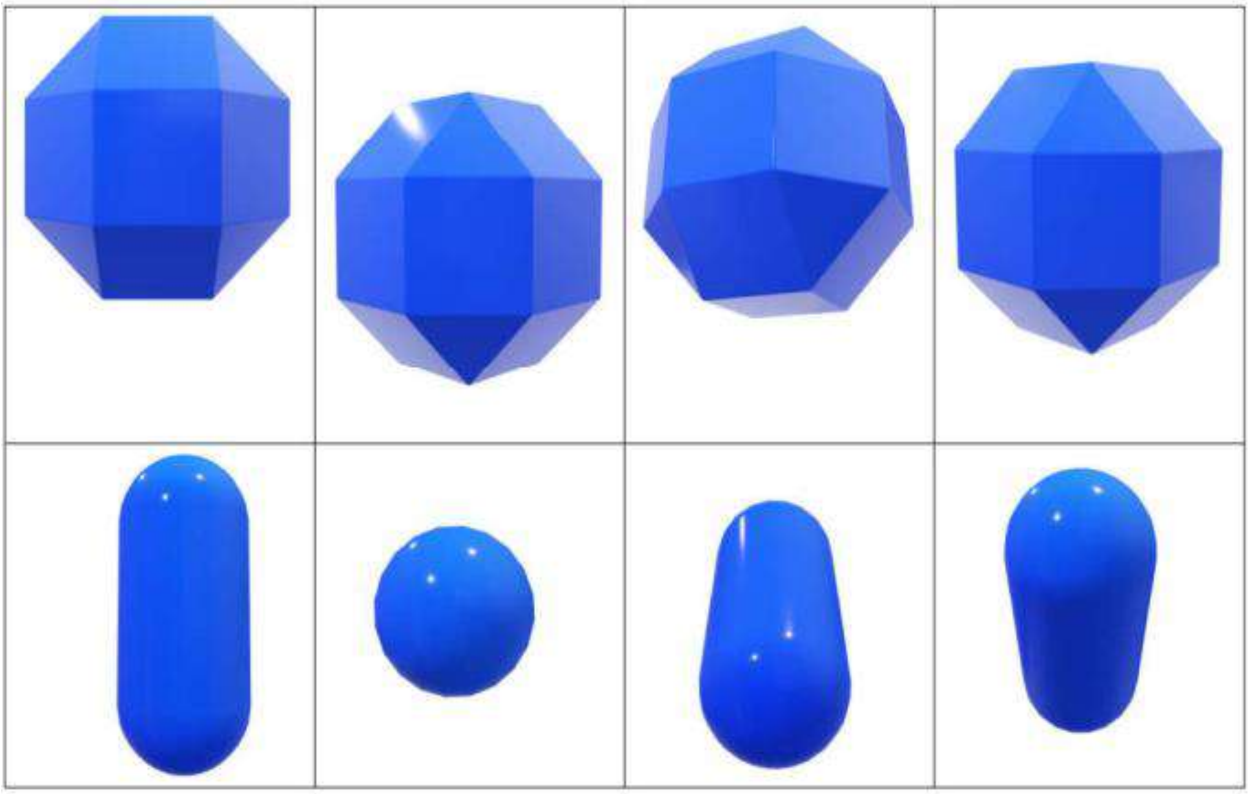
3D Shapes

Here are 11 different 3D Shapes. Which has been observed from different faces and try to analyze its basic Euclidian Shapes.









Those 3D shapes are the conjunction of different Shapes Rectangle, Circle and triangle apparently. But as rectangle are a form of Parallelogram hence,

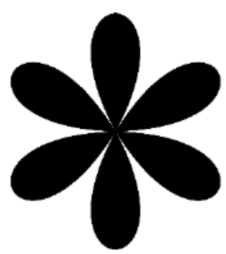
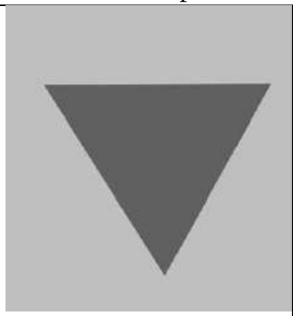
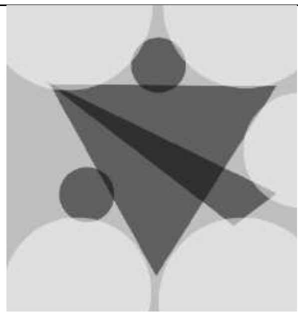
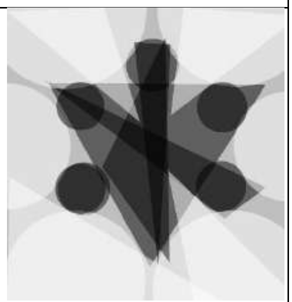
∴area of Parallelogram=2 ×area of forming triangle (Euclide et al.)

Hence it can be Said that, all of the shapes are formed with rearrangements of Triangle and Circle. Moreover, these 3D shapes are also building Shapes in 3D geometry, who further rearranged and assimilated to form some new pattern and shapes.

Analyses of some Natural Object:

Here Six objects has been Selected from different sources of nature with randomly. and using Computer Software to generate the geometric Shapes, patterns by using only two Geometric pattern which are Circle and Triangle to find out though there are infinite number of appearances in the nature is there any fundamental geometric Pattern. Moreover by multiplying number of geometric patterns the quality of the image has been improved when the occupation of the space has been completed to attain ≅ 100% similarity in appearance compared with real photographed image.

Flower petals distribution Pattern: to recreate the flower petals distribution both triangle and circle has been used.

Flower petals distribution Pattern			
			
Real image. Figure 1	No. of Shape 1	No. of Shape 10	No. of Shape 20

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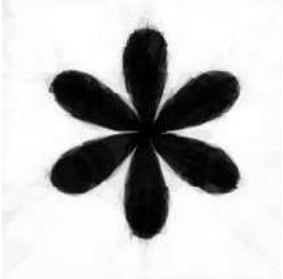
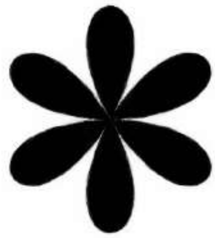
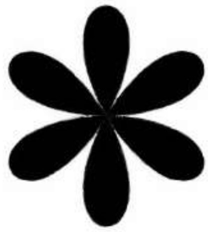
<tr><td></td><td></td><td></td><td></td></tr>
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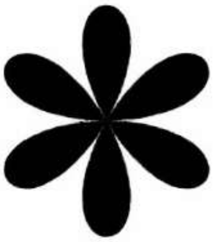
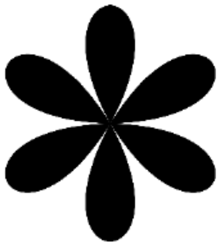
<tr><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td colspan="2">No of Shape 2000</td><td colspan="2">Real image. Figure 1</td></tr>
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Flower petals Pattern: to recreate the flower petals to create this only triangle has been used.

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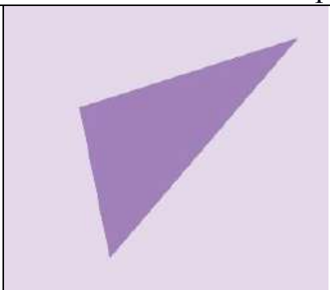
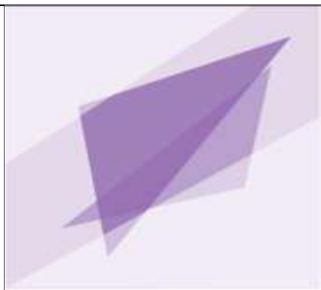
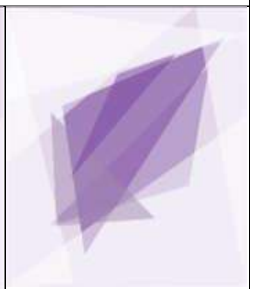
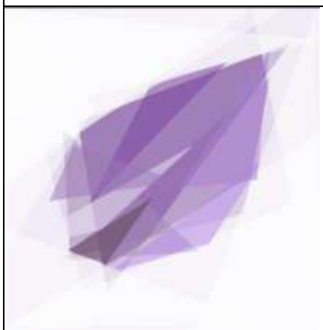
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>The Real Image. Figure 2</td><td>No. of triangle 1</td><td>No. of triangle 5</td><td>No. of triangle 10</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
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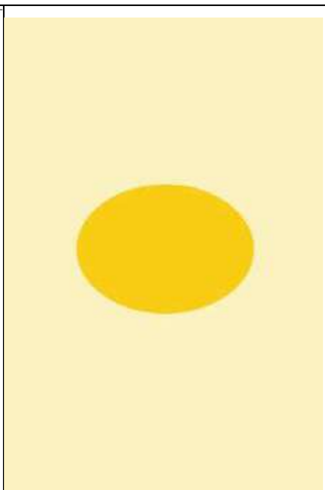
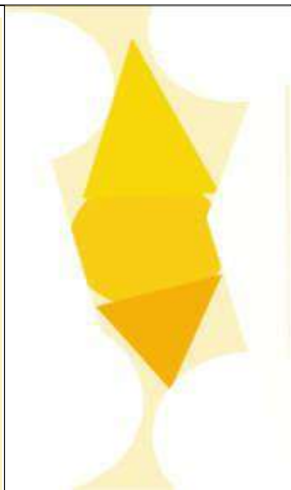
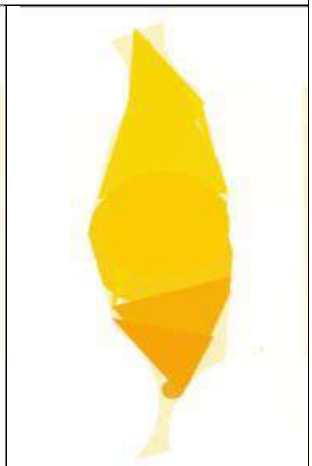
No. of triangle 2000	No.of triangle 5000	No. of triangle 10000

	
Final output with 22,000 triangle with 98% accuracy.	Real Image

Flower petals distribution Pattern: to recreate the flower petals distribution both triangle and circle has been used.

The Flower buds			
			
The Real Image. Figure 3	No. of Shape 1	No. of Shape 10	No. of Shape 20

			
No. of Shape 50	No. of Shape 100	No. of Shape 1000	No. of Shape 5000
			
No. of Shape 10000	No. of Shape 15,700 Similarity accuracy 98.62	The Real Image. Figure 3	

Butterfly wings Pattern: to recreate the Butterfly wings Pattern both triangle and circle has been used

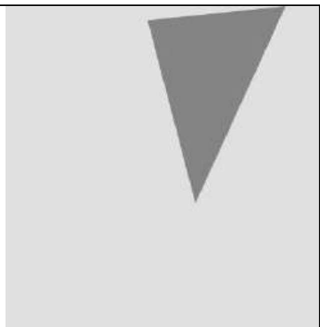
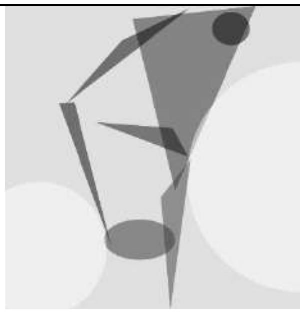
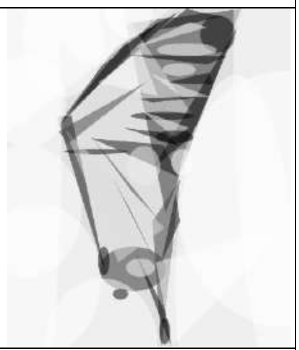
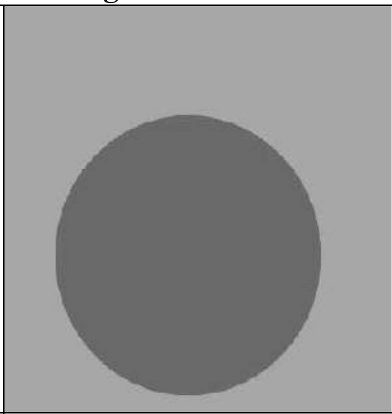
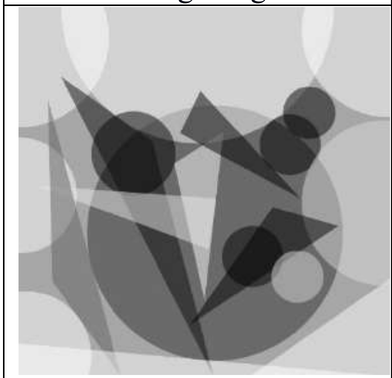
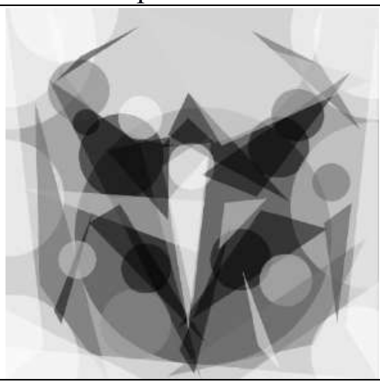
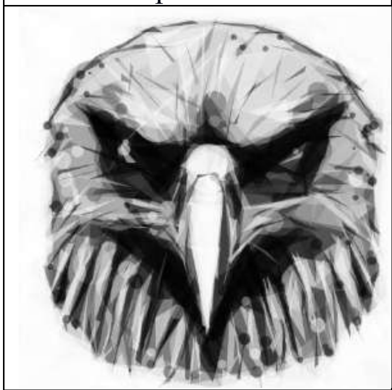
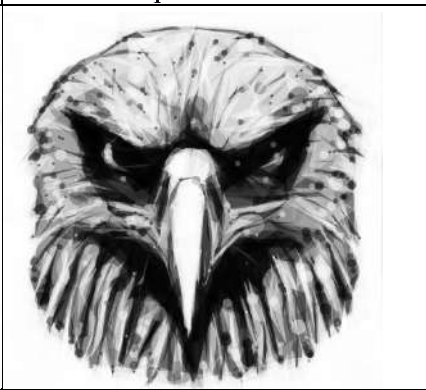
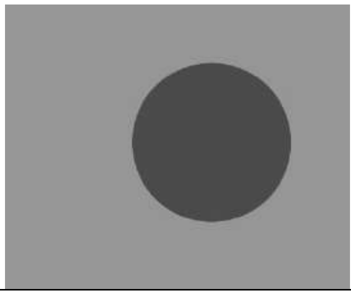
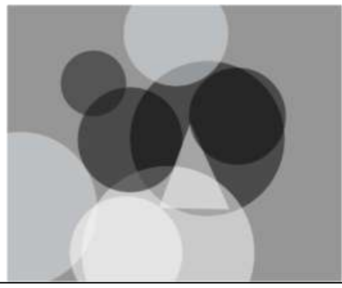
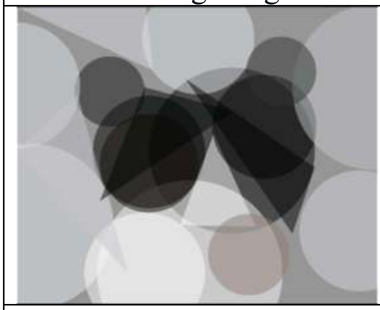
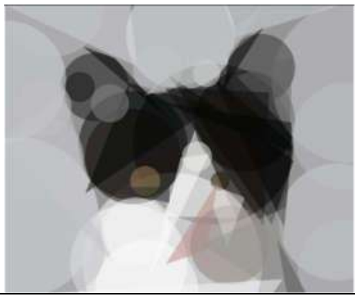
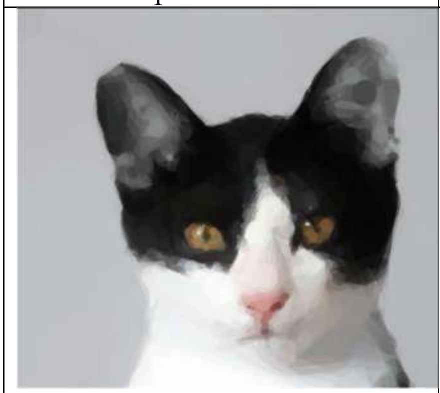
The butterfly Wings			
			
The Real Image.	No. of Shape 1	No. of Shape 10	No. of Shape 50

Figure 4			
			
No. of Shape 100	No. of Shape 500	No. of Shape 1000	No. of Shape 10000
			
No. of Shape 15000	No. of Shape 20000	No. of Shape 25000	No. of Shape 30000
			
No. of Shape 32000		The Real Image. Figure 5	

The eagle face Pattern: : to recreate the Butterfly wings Pattern both triangle and circle has been used.

The Eagle Face		
		
The Real Image. Figure 5	No. of Shape 1	No. of Shape 10
		
No. of Shape 20	No. of Shape 50	No. of Shape 100
		
No. of Shape 500	No. of Shape 1000	No. of Shape 5000
		
No. of Shape 10000	No. of Shape 10700 with 95% accuracy	The Real Image. Figure 5

The Cat face Pattern: : to recreate the Butterfly wings Pattern both triangle and circle has been used.

		
The Real Image. Figure 1	No. of Shape 1	No. of Shape 10
		
No. of Shape 20	No. of Shape 50	No. of Shape 100
		
No. of Shape 500	No. of Shape 1000	No. of Shape 5000
		
No. of Shape 10000	No. of Shape 15000	

	
No. of Shape 17907 With 97% Accuracy	The Real Image. Figure 1

Postulates:
Hence, from the above-mentioned studies, the Geometrical postulates can now be said to be:
Circle: A circle is an expression of a point that is two-dimensional and continuous. To analyse the fundamental relation between a Circle and a point, examine the Angles. Moreover, when the Angles are continuous and unidirectionally progressed, the accumulated form of its path is expressed as a complete circle. Therefore, a Circle is a kind of geometrical expressions who have a continuous connection with its centre/ origin.
Triangle: When a pattern or Shape does not follow the root or origin of its parent Shape, instead follows the expression of the Shape, different patterns are originated. Among them, the most fundamental Shape is the Triangle.
However, there are two fundamental geometrical patterns in this universe. One is a circle, which is a continuous progression by maintaining its relation with the point of origin, and the Second one is a triangle. Who does not need to follow the origin but follow the path of the circumference of the circle, or in other words, only the appearance of the prior Shape.

Conclusion: The Earth is composed of an infinite number of objects, but the formation structure of them is limited to two shapes: circle and Triangle. One who strictly follows the root of origin, and another one follows the expression of its parental Shape, not the origin of the parental Shape.

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Image Resources:
1. Fundamental Geometric Pictures are digitally illustrated by Chorel Draw 11.
2. Figure 1,2,3 downloaded from google
3. Vector recreation of 1,2,3 Geometrize 1.0.1
4. 3D Shapes are downloaded from Microsoft Webstore.