FUZZY COLORING OF FUZZY PATHS USING MATLAB

M.G. Karunambigai[1], Jessalet Ann Mathew[2*]

[1,2]Department of Mathematics, Sri Vasavi College, Erode-638 316, Tamil Nadu, India.*
***Email:** [1]karunsvc@yahoo.in*

***Corresponding author :** *Jessalet Ann Mathew.*
***Email:** *jessaletannmathew@gmail.com*

Abstract
In this paper, MATLAB algorithms are developed to calculate the chromatic number of fuzzy paths and their middle graph and it generates the diagrammatic representations of these fuzzy graphs with proper fuzzy coloring.

Keywords: *Fuzzy Coloring, Chromatic Number, Fuzzy Path, Middle Graph, MATLAB.*

1. Introduction

Fuzzy coloring represents an important generalization of classical graph coloring within fuzzy graph theory, providing a flexible framework for modeling and solving problems in systems where uncertainty or imprecision exists in relationships. It plays a crucial role in resource allocation and task scheduling, ensuring conflict-free schedules and optimal resource utilization [1]. Since many real-world problems can be expressed as coloring problems, fuzzy coloring has emerged as a key area of study in graph theory. This idea was first introduced by Susana Munoz et al. [2], who proposed a method for coloring fuzzy graphs with a crisp vertex set and a fuzzy edge set and further defined the chromatic number of a fuzzy graph as a fuzzy subset of its vertex set. Subsequently, Eslahchi and Onagh [3] refined this approach by considering strong adjacency between vertices in determining the chromatic number. Later, Sovan Samanta et al. [4] extended the concept by introducing fuzzy colors, developing an alternative technique in which colors are assigned according to the strength of the edges incident on each vertex. This approach provides a more refined and application-oriented method for coloring fuzzy graphs. Furthermore, in 2024, we determined the chromatic number of certain families of fuzzy graphs using fuzzy colors based on the strength of an edge incident to a vertex and derived various properties of fuzzy coloring [5]. We also computed the chromatic number of some related graphs of fuzzy paths, fuzzy cycles and homogeneous fuzzy caterpillars. (refer [6],[7],[8]).
MATLAB was originally created to simplify the solution of systems of linear equations and eigenvalue problems [9]. Since its development in 1984, it has grown from a matrix-oriented tool into a high-performance technical computing language widely adopted in academia and industry. Its design integrates computation, visualization and programming in a single environment, enabling efficient implementation of complex algorithms and rapid prototyping of ideas. Unlike conventional languages such as C and FORTRAN, MATLAB is based on arrays as the fundamental data element, eliminating the need for explicit dimensioning [10]. Over time, it has expanded to include sophisticated data structures, built-in editing and debugging tools and support for object-oriented programming, thereby positioning itself as both a research and teaching tool [11]. Furthermore, its toolboxes extend MATLAB's functionality across diverse domains such as signal processing, control theory, symbolic computation and optimization, making it an indispensable resource for advancing science and engineering. In the present study, we are trying to develop a MATLAB coding to determine the chromatic number of fuzzy paths and its middle graph using the proper fuzzy coloring procedure.

The organization of this paper is as follows. An overview of fuzzy coloring and MATLAB is presented in Section 1. Section 2 outlines the fundamental definitions and preliminary concepts relevant in this study. In Section 3, we develop MATLAB coding to determine the chromatic number of a fuzzy path by using the proper fuzzy coloring procedure. In Section 4, focuses on the implementation of MATLAB code to compute the chromatic number of middle graph of a fuzzy path by using the proper fuzzy coloring procedure.

2. Preliminaries

This section outlines some definitions of fuzzy graph theory and fuzzy coloring, which serve as the foundation for evaluating the chromatic number of fuzzy paths and its middle graph within the MATLAB environment.

Definition 2.1. [12] A fuzzy graph $G = (V, \sigma, \mu)$ is a pair of functions (σ, μ), where $\sigma : V \to [0, 1]$ is a fuzzy subset of a non-empty set V and $\mu : V \to [0, 1]$ is a symmetric fuzzy relation on σ, such that the relation $\mu(v_i, v_j) \leq \sigma(v_i) \wedge \sigma(v_j)$ is satisfied for all $v_i, v_j \in V$ and $(v_i, v_j) \in E \subset V \times V$.
Here, $\sigma(v_i)$ denote the degree of membership of the vertex v_i and $\mu(v_i, v_j)$ denotes the degree of membership of the edge relation $e_{ij} = (v_i, v_j)$ on $V \times V$.

Note : In this paper, we denote $\sigma(v_i) \wedge \sigma(v_j) = min\{\sigma(v_i), \sigma(v_j)\}$ and $\sigma(v_i) \vee \sigma(v_j) = max\{\sigma(v_i), \sigma(v_j)\}$.

Definition 2.2. [13] Let $G = (V, \sigma, \mu)$ be a fuzzy graph with underlying crisp graph G^*. A fuzzy path P_n in G is a sequence of distinct vertices $v_0, v_1, \dots, v_n$ such that $\mu(v_{i-1}, v_i) > 0, 1 \leq i \leq n$. Here $n \geq 1$ is called the length of the path P_n.

Definition 2.3. [4] Let $G = (V, \sigma, \mu)$ be a fuzzy graph and an edge $e = (v_i, v_j) \in G$ is called **strong** if $\frac{1}{2}\{\sigma(v_i) \wedge \sigma(v_j)\} \leq \mu(v_i, v_j)$ and it is called **weak** otherwise.

Definition 2.4. [4] Let $G = (V, \sigma, \mu)$ be a fuzzy graph and the **strength of an edge** $(v_i, v_j) \in G$ is denoted by,

$$I(v_i, v_j) = \frac{\mu(v_i, v_j)}{\sigma(v_i) \wedge \sigma(v_j)}.$$

Definition 2.5. [14] A fuzzy graph $G = (V, \sigma, \mu)$ is called a strong fuzzy graph if each edge in G is a strong edge.

Definition 2.6. [5] Let $G = (V, \sigma, \mu)$ be a fuzzy graph. Fuzzy coloring is an assignment of basic or fuzzy colors to the vertices of a fuzzy graph G and it is proper,

(i) if two vertices are connected by a strong edge, then they either have different basic or fuzzy colors(if necessary), or one vertex can have a basic color and the other can have a fuzzy color corresponding to different basic color.

(ii) if two vertices are connected by a weak edge, then they either have same or different fuzzy colors, or one vertex can have a basic color and other can have a fuzzy color corresponding to the same basic color.

Definition 2.7. [5] Let $G = (V, \sigma, \mu)$ be a fuzzy graph. Perfect fuzzy coloring (optimal fuzzy coloring) is an assignment of minimum number of colors (basic or fuzzy) for a proper fuzzy coloring of G.

Definition 2.8. [5] Let $G = (V, \sigma, \mu)$ be a fuzzy graph. The minimum number of colors (basic or fuzzy) needed for a proper fuzzy coloring of G is called the chromatic number of G and is denoted by $\chi_f(G)$.

Definition 2.9. [6] The middle graph $M_f(G)(V_M, \sigma_M, \mu_M)$ of a fuzzy graph $G(V, \sigma, \mu)$ is a fuzzy graph with underlying crisp graph $M(G)(V_M, E_M)$, with the vertex set $V_M = V \cup V_{ij}$ where $V = \{v_i \mid v_i \in V\}$ and $V_{ij} = \{v_{ij} \,\forall\, (v_i, v_j) \in E\}$ and $v(M_f(G)) = n + 1 + n = 2n + 1$ and the edge set

$$E_M = \begin{cases} (v_{ij}, v_i), (v_{ij}, v_j) & \forall\, i \text{ and } j, \\ (v_{ij}, v_{rs}) & \text{if the edges } (v_i, v_j) \text{ and } (v_r, v_s) \text{ are adjacent in } G. \end{cases}$$

Then, $\sigma_M(v_i) = \sigma(v_i)$ *if* $v_i \in V, 0 \leq i \leq n$,

$\sigma_M(v_{ij}) = \mu(v_i, v_j)$ *if* $(v_i, v_j) \in E \;\forall\; i$ *and* j,

$\mu_M(v_{ij}, v_{rs}) = \mu(v_i, v_j) \wedge \mu(v_r, v_s)$ if the edges (v_i, v_j) and (v_r, v_s) are adjacent in G,

and $\mu_M(v_i, v_{ij}) = \mu_M(v_j, v_{ij}) = \mu(v_i, v_j) \;\forall\; i$ *and* j.

Lemma 2.1. [5] *Let P_n be a fuzzy path of length n. If all the edges are strong in P_n, then* $\chi_f(P_n)$= 2.

Lemma 2.2. [5] *Let P_n be a fuzzy path of length n. If all the edges are strong in P_n, then* $\chi_f(P_n)$= 2.

Theorem 2.1. [5] *Let P_n be a fuzzy path of length n. If atleast one edge is strong in P_n, then $\chi_f(P_n) = 2$.*

Theorem 2.2.[6] *If $M_f(P_n)$ is a strong fuzzy graph, then $\chi_f(M_f(P_n)) = 3$.*

Theorem 2.3. [6] $\chi_f(G) \geq max\,\{\chi_f(G_i) : 1 \leq i \leq k\}$, *where* $G = G_1 \cup G_2 \cup \ldots \cup G_k$ *and* $G_i, 1 \leq i \leq k$ *are fuzzy graphs.*

Corollary 2.3.1. [6] $\chi_f(G) \geq max\,\{\chi_f(G_i) : 1 \leq i \leq k\}$, *where* $G = G_1 \oplus G_2 \oplus \ldots \oplus G_k$ *and* $G_i, 1 \leq i \leq k$ *are edge disjoint fuzzy graphs.*

3. Fuzzy Coloring of Fuzzy Paths Using MATLAB

This study implements the proper fuzzy coloring of fuzzy path P_n using MATLAB. The procedure is divided into five main steps : Input of number of vertices and edges with membership values, Edge classification, Proper Fuzzy coloring, Computation of the chromatic number of the fuzzy path and Visualization of fuzzy path with proper fuzzy coloring.

Step 1: Input of Membership Values

Let P_n be a fuzzy path with n vertices and $n - 1$ edges. The membership values of the vertices are denoted by $\sigma(v_i)$, where $0 \leq i \leq n$ and the membership values of the edges are denoted by $\mu(v_i, v_{i+1})$, where $0 \leq i \leq n - 1$. These values are taken as input from the user.

Step 2: Edge Classification

Each edge is classified as strong or weak using the following rule: If

$$\frac{1}{2}\{\sigma(v_i) \wedge \sigma(v_j)\} \leq \mu(v_i, v_j),$$

Then the edge (v_i, v_{i+1}) is **strong**; otherwise, it is **weak**. The classification results are displayed for all edges in the fuzzy path.

Step 3: Proper Fuzzy Coloring of Vertices

Coloring proceeds sequentially from vertex v_0 to v_n under the following rules:

1. Weak Edge : Assign a basic color with full intensity to a vertex and assign the adjacent vertex the same color as its preceding vertex but with diluted intensity calculated as,
$$Intensity = 1 - \frac{\mu(v_i, v_j)}{\sigma(v_i) \wedge \sigma(v_j)}.$$
2. Strong Edge : Each pair of adjacent vertices must be assigned different basic colors with full intensity.

Step 4: Computation of the Chromatic Number

The chromatic number $\chi_f(P_n)$ is computed. This value denotes the minimum number of colors (basic or fuzzy) required for the proper fuzzy coloring.

Step 5: Visualization

The fuzzy path P_n is visualized in MATLAB:

1. Vertices are represented by circular nodes labeled with their names, membership values and color-intensity pairs.
2. Edges are drawn as black lines and their membership value μ is displayed at the midpoint.

This visualization provides a clear and intuitive representation of proper fuzzy coloring with the chromatic number.

Example 1. Figure 1 presents the MATLAB output for the proper fuzzy coloring of the fuzzy path P_4, in which all edges are weak. The vertices v_0, v_1, v_2, v_3, v_4 have membership values $0.9, 0.8, 0.7, 0.6, 0.5$, while the edges $(v_0, v_1), (v_1, v_2), (v_2, v_3), (v_3, v_4)$ have membership values $0.39, 0.34, 0.23, 0.24$.

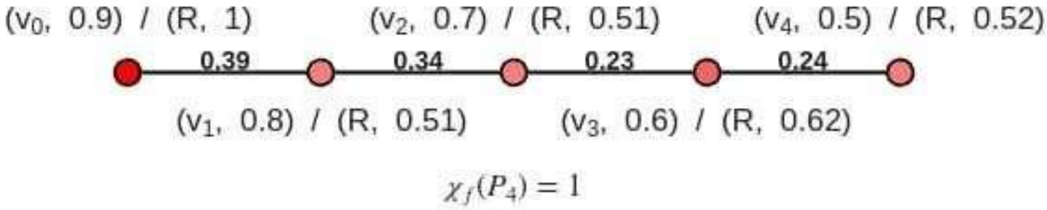

Figure 1. Fuzzy coloring of fuzzy path P_4, when all the edges are weak.

Example 2. Figure 2 presents the MATLAB output for the proper fuzzy coloring of the fuzzy path P_4, in which all edges are strong. The vertices v_0, v_1, v_2, v_3, v_4 have membership values $0.9, 0.8, 0.7, 0.6,\ 0.5$, while the edges $(v_0, v_1), (v_1, v_2), (v_2, v_3), (v_3, v_4)$ have membership values $0.5, 0.5, 0.5, 0.5$.

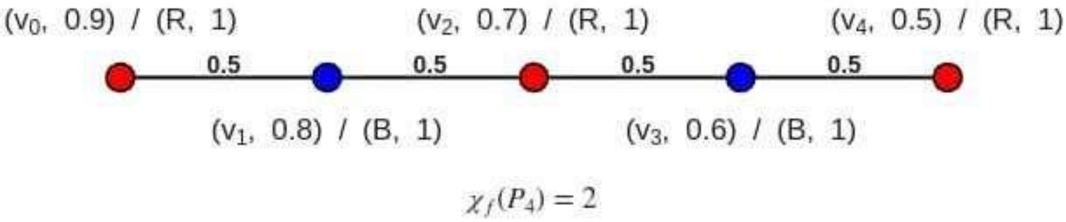

Figure 2. Fuzzy coloring of fuzzy path P_4, when all the edges are strong.

Example 3. Figure 3 presents the MATLAB output for the proper fuzzy coloring of the fuzzy path P_4 , in which atleast one edge is strong. The vertices v_0, v_1, v_2, v_3, v_4 have membership values $0.9, 0.8, 0.7,\ 0.6, 0.5$, while the edges $(v_0, v_1), (v_1, v_2), (v_2, v_3), (v_3, v_4)$ have membership values $0.4, 0.34, 0.4,\ 0.24$.

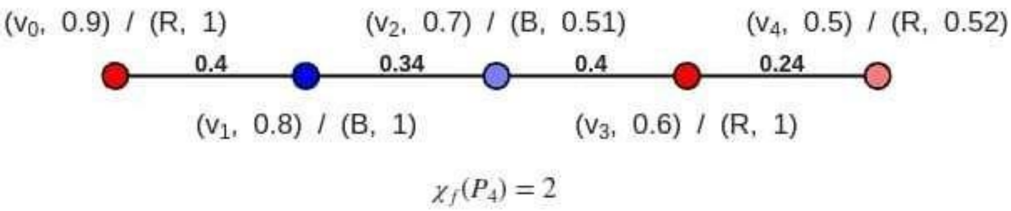

Figure 3. Fuzzy coloring of fuzzy path P_4 with atleast one edge is strong.

4. Fuzzy Coloring of Middle Graph of Fuzzy Paths Using MATLAB

This study implements the proper fuzzy coloring of the middle graph of fuzzy paths P_n using MATLAB. The procedure is divided into six main steps: Input of the number of vertices and edges along with their membership values, Construction of the vertex and edge sets of the fuzzy middle graph, Classification of edges into strong and weak types, Proper fuzzy coloring of the vertices, Computation of the chromatic number and Visualization of the middle graph of P_n with proper fuzzy coloring.

Step 1: Input of Membership Values
Let P_n be a fuzzy path with n vertices and $n-1$ edges. The membership values of the vertices are denoted by $\sigma(v_i)$, where $0 \leq i \leq n$ and the membership values of the edges are denoted by $\mu(v_i, v_{i+1})$, where $0 \leq i \leq n-1$. These values are taken as input from the user.
Step 2: Construction of the Vertex Set
The middle graph $M_f(P_n)$ is constructed by two categories of vertices:

1. All vertices of the fuzzy path P_n, namely $v_0, v_1, \dots, v_n$, each carrying its corresponding membership value $\sigma(v_i)$.
2. For each edge $(v_i, v_{i+1}) \in P_n$, a new vertex v_{ii+1} is added. The membership value of this vertex is defined as $\sigma(v_{ii+1}) = \mu(v_i, v_{i+1})$.

Thus, the vertex set is given by, $V\left(M_f(P_n)\right) = \{v_0, v_1, \dots, v_n\} \cup \{v_{01}, v_{12}, \dots, v_{n-1n}\}$.

Step 3: Construction of the Edge Set
There are two types of edges in $M_f(P_n)$:

1. For each edge (v_i, v_{i+1}) in P_n, the vertex v_{ii+1} is connected to both v_i and v_{i+1} with membership value $\mu(v_i, v_{ii+1}) = \mu(v_{i+1}, v_{ii+1}) = \mu(v_i, v_{i+1})$.
2. If two consecutive edges (v_i, v_{i+1}) and (v_{i+1}, v_{i+2}) exist in P_n, then the vertices v_{ii+1} and v_{i+1i+2} are connected with membership value

$$\mu(v_{ii+1}, v_{i+1i+2}) = min\{\mu(v_i, v_{i+1}), \mu(v_{i+1}, v_{i+2})\}.$$

Step 4: Edge Classification

Each edge (v_i, v_j) of $M_f(P_n)$ is classified as either strong or weak based on the following criterion:

$$\frac{1}{2}\{\sigma(v_i) \wedge \sigma(v_j)\} \leq \mu(v_i, v_j),$$

If the condition is satisfied, the edge (v_i, v_j) is **strong**. Otherwise, the edge is **weak**.

Step 5: Coloring of the Middle Graph and Computation of the Chromatic Number
Since the middle graph $M_f(P_n)$ is a strong fuzzy graph, the adjacent vertex must be assigned different basic colors with full intensity. Also, the chromatic number $\chi_f(M_f(P_n))$ is computed. This value reflects the minimum number of colors (basic or fuzzy) required for the proper fuzzy coloring.

Step 6: Visualization
Finally, the graph is visualized such that

1. Vertices are represented by circular nodes labeled with their names, membership values and color-intensity pairs.
2. Edges are drawn between vertices with their membership values μ displayed at the midpoint.

This visualization provide a clear and intuitive representation of proper fuzzy coloring with the chromatic number.

Example 4. Figure 4 presents the MATLAB output for the proper fuzzy coloring of middle graph of fuzzy path P_2. The vertices v_0, v_1, v_2 have membership values $0.9, 0.8, 0.7$, while the edges $(v_0, v_1), (v_1, v_2)$ have membership values $0.4, 0.2$.

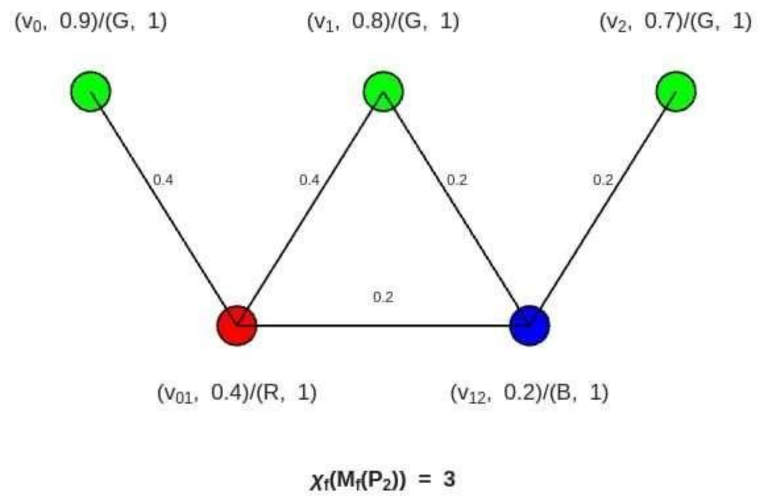

Figure 4. Fuzzy coloring of middle graph (P_2).

5. References

[1] Kannan, M., Sathiragavan, M., Nivetha, P., Sankar, K., & Gurjar, J. (2024). Graph coloring techniques in scheduling and resource allocation. Journal of Nonlinear Analysis and Optimization, 15(2).

[2] Munoz, S., Ortuno, M. T., Ramirez, J., & Yanez, J. (2005). Coloring fuzzy graphs. Omega, 33(3), 211–221.

[3] Eslahchi, C., & Onagh, B. N. (2006). Vertex-strength of fuzzy graphs. International Journal of Mathematics and Mathematical Sciences, 2006(1), 43614.

[4] Samanta, S., Pramanik, T., & Pal, M. (2016). Fuzzy colouring of fuzzy graphs. Afrika Matematika, 27(1–2), 37–50.

[5] Karunambigai, M. G., & Mathew, J. A. (2024). The chromatic number of certain families of fuzzy graphs. Cuestiones Fisioterapia, 53(3), 5153–5268.

[6] Karunambigai, M. G., & Mathew, J. A. (2025). The chromatic number of fuzzy paths and its related graphs. International Journal of Environmental Science,11(18s), 1858–1869.

[7] Karunambigai, M. G., & Mathew, J. A. (2025). The chromatic number of fuzzy cycles and its related graphs. International Journal of Environmental Science,11(20s), 3697–3709.

[8] Karunambigai, M. G., & Mathew, J. A. (2025). The chromatic number of homogeneous fuzzy caterpillars and its related graphs. International Journal of Applied Mathematics, 38(3s), 928–942.

[9] Houcque, D. (2005). Introduction to MATLAB for engineering students (p. 1). Northwestern University.

[10] Moler, C., & Little, J. (2020). A history of MATLAB. Proceedings of the ACM on Programming Languages, 4(HOPL), 1–83.

[11] Abdul Majid, M., Huneiti, Z. A., Balachandran, W., & Balarabe, Y. (2013). MATLAB as a teaching and learning tool for mathematics: A literature review. International Journal of Arts & Sciences, 6(1), 205–221.

[12] Mathew, S., Mordeson, J. N., & Malik, D. S. (2018). Fuzzy graph theory. Springer International Publishing.

[13] Pal, M., Samanta, S., & Ghorai, G. (2020). Fuzzy cut vertices and fuzzy trees. In Modern trends in fuzzy graph theory (pp. 115–124). Springer.

[14] Gong, Z., & Zhang, J. (2022). Chromatic number of fuzzy graphs: Operations, fuzzy graph coloring and applications. Axioms, 11(12), 697.