Scheme of Derived Moduli Problem to the "quantum" version of an algebra symT.

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In this research, will be obtained the ∞ -category analogous more algebraic structures like commutative rings to obtain the "quantum" version of an algebra symT, through consider a scheme to a moduli problem on a field k (class field) of all equivalences that are satisfied in the context of the *moduli schemes* and "CRings". Also is given a short classification of derived moduli problems and their elements in moduli stacks.

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1 Introduction

We consider the extension of functors between derived categories using tools of Yoneda embeddings and Grothendieck's "functor of points" considering the moduli space X, as equivalent to specifying of the functor

$$R \to X(R) = \mathrm{Hom}(\mathrm{Spec}R, X) \tag{1.1}$$

For other way, let k, be a field. The category Chain$_k$, of chain complexes over k, admits a symmetric monoidal structure, given by the usual tensor product of chain complexes. A commutative algebra in the category Chain$_k$, is called a commutative differential graded algebra over k. The functor $R \to X(R)$, is symmetric monoidal, and determines a functor

$$\varphi : \mathrm{CAlg}(\mathrm{Chain}_k) \to \mathrm{CAlg}(\mathrm{Mod}_k) \cong \mathrm{CAlg}_k, \tag{1.2}$$

We say that a morphism $f : A_\bullet \to B_\bullet$, in CAlg_k^{dg}, is a quasi-isomorphism if it induces a quasi-isomorphism between the underlying chain complexes of $A_\bullet$, and B$_\bullet$. The functor φ, carries every quasi-isomorphism of commutative differential graded algebras to an equivalence in CAlg$_k$.

Here can appear an extended version of Penrose transform in the context of the moduli problems, that is to say, considering the schemes in the context Moduli$_k$, can be constructed a functor that comes from a sufficiently generalized Penrose transform such that the objects induced in an augmented algebra corresponds to geometrical objects of the functor Chain$_k$, (in this case we need that these objects are CW- complexes) applied in the context of the vector bundles that come from the commutative differential graded algebra over k. If k, is a field of characteristic zero, then φ, induces an equivalence

$$\mathrm{CAlg}(\mathrm{Chain}_k)[W^{-1}] \cong \mathrm{CAlg}_k, \tag{1.3}$$

where W, is the collection of quasi-isomorphisms, that in the context of the ∞- categories of E_∞- algebras over k, will be the obtained from the ordinary categories of commutative differential graded k- algebras by formally inverting the collection of quasi-isomorphisms. The medullar functors of these transformations are functores obtained through the Yoneda embedding [2]. An classification of these functors is obtained using the equivalences between objects in three levels: the formal moduli problems, the integral transforms and the Zuckerman functors,where this last result useful in the representation problem of Lie algebras in the geometrical context planted by the Langlands program with their geometrical correspondences.

Likewise, are important the considerations on the affine algebraic varieties that can be used to define algebras of objects that can transit from a CAlg_k, as could be an $\mathbb{C}-$ algebra until an ∞- algebra in Alg^n_{aug} to our extension to a "quantum version" of an algebra Sym. This last can be got by a functor Ext, considering the moduli problems between objects of an algebra, which has been realized using commutative rings extended by a Yoneda algebra.

Then for the Yoneda embedding [1], considering the Yoneda algebra construction foreseen in [2] and considering an ∞- algebra in $\mathrm{Alg}^{(n)}_{aug}$, we can give an isomorphism between ∞- categories and E_∞- rings of a ∞- category of spaces as $\mathrm{Spec}_H \mathrm{symT}_0 A, \forall A \in OP_{L_G}(\Sigma)$, with Σ, a smooth projective complex curve, which is the equivalence of two categories. This ∞- category of spaces can be extended to the non-commutative geometry and deformed categories. This gives a scheme of the derived moduli problem to the quantum version wanted. To it, is necessary consider the foreseen in [3] on functoriality of moduli problems.

2 Moduli Problems for Commutative Rings.

Let $\mathcal{R}$, be denote the category of commutative rings and $\mathcal{S}$, the category of sets. Doing extensive the use of Grothendieck's "functor of points" philosophy: that is, we will identify a geometric object X (such as a scheme) with the functor $\mathcal{R} \to \mathcal{S}$, represented by X, given by the formula $R \to \mathrm{Hom}(\mathrm{Spec}R, X)$.

We consider the following example.

Example 2. 1. We fix an integer $n \geqslant 0$. We define a functor $X : \mathcal{R} \to \mathcal{S}$, by letting $F(R)$, denote the set of all submodules $M \subseteq R^{n+1}$, such that the quotient R^{n+1}/M, is a projective R- module of rank n(from which it follows that M , is a projective R- module of rank 1). The functor X , is not representable by a commutative ring. However, it is representable in the large category Sch, of schemes. That is, for any commutative ring R, we have a canonical bijection

$$X(R) \cong \mathrm{Hom}_{\mathrm{Sch}}(\mathrm{Spec}\ R, \mathrm{P}^n), \tag{2.1}$$

where $P^n \cong \mathrm{Proj}\mathbf{Z}[\chi_0, ..., \chi_n]$, denotes projective space of dimension n.

For our proposes, the action of a functor $X : \mathcal{R} \to S$, is too restrictive. We often want to study moduli problems X, which assign to a commutative ring R, some class of geometric objects which depend on R. The trouble is that this collection of geometrical objects is naturally organized into a category, rather than a set. This motivates the following definition:

Def. 2. 1. Let Gpd, denote the collection of grupoids, that is to say categories in which every morphism is an isomorphism. We regard Gpd, as a 2-category, that is to say, morphisms are given by functors between grupoids and 2- morphisms are given by natural transformations (which are automatically invertible). A classical moduli problem is a functor $X : \mathcal{R} \to \mathrm{Gpd}$.

Every set S, can be regarded as a grupoid by setting

$$\mathrm{Hom}_S(x, y) = \begin{Bmatrix} (id_x), \text{if } \ x = y \\ \varnothing, \text{if } \ x \neq y \end{Bmatrix} \tag{2.2}$$

This construction allows us to identify the category S, with a full subcategory of the 2- category Gpd. In particular, every functor $X : \mathcal{R} \to S$, can be identified with a classical moduli problem in the sense of the *definition 2. 1.*

Example 2.2. For every commutative ring R, let $X(R)$, be the category of elliptic curves $E \to \mathrm{Spec}\ R$ (morphisms in the category $X(R)$, are given by isomorphisms of elliptic curves). Then F, determines a functor $\mathcal{R} \to \mathrm{Gpd}$, and can therefore be regarded as a moduli problem in the sense of the *definition 2. 1.* This moduli problem cannot be represented by a commutative ring or even by a scheme, that is to say, for any scheme Y, the space $\mathrm{Hom}_{\mathrm{Sch}}(\mathrm{Spec}R, Y)$, is a set. In particular if we remember the space $\mathrm{Hom}_{\mathrm{Sch}}(\mathrm{Spec}R, Y)$ as a grupoid, every object has a trivial automorphism group. In contrast, every object of $X(R)$, has a non-trivial automorphism group, that is to say, every elliptic curve admits a nontrivial automorphism, given by multiplication by -1.

Nevertheless, the moduli problem, is representable if we work not in the category of schemes but yes in the larger 2- category $\mathrm{St}_{\mathrm{DM}}$, of *Deligne-Mumford stacks*. More precisely, there exists a Deligne-Mumford stack $\mathcal{M}_{\mathrm{E11}}$ (*the moduli stack of elliptic curves*) for which there is a canonical equivalence of categories

$$X(R) \cong \mathrm{Hom}_{\mathrm{St}_{\mathrm{DM}}}(\mathrm{Spec}R, \mathcal{M}_{E11}), \tag{2.3}$$

for every commutative ring R.

3 Rings and their Spectrums.

Let $\mathscr{C}$, be the category of all topological spaces and let hom_{π} , be the collection of weak homotopy equivalences. We will refer to $\mathscr{C}[W^{-1}]$, as the ∞-category of spaces and denote it by $\mathscr{S}$. We describe the object of $\mathscr{C}$, as the CW complexes [4], and whose property between CW complexes is:

(a) The objects of $\mathscr{C}$, are CW complexes.
(b) For every pair of CW complexes X, and Y, we let $\mathrm{Hom}_{\mathscr{C}}(X, Y)$, denote the space of continuous maps from X, to Y, (endowed with the compact-open topology).

The role of $\mathscr{C}$, in the theory of ∞- categories is analogous to the ordinary category of sets in classical category theory. For example, for any ∞-category $\mathscr{C}$, one can define a *Yoneda embedding* [1]

$$j : \mathscr{C} \to \mathrm{Fun}(\mathscr{C}^{top}, \mathscr{S}), \tag{3.1}$$

given explicitly by

$$j(C)D = \mathrm{Hom}_{\mathscr{C}}(D, C) \in \mathscr{S}, \tag{3.2}$$

In this research, we are interested in the ∞-category analogous more algebraic structures like commutative rings (see the Table 1).

Classical Structure	∞- categorical Analogue
Set	Topological Space
Category	∞- category
Abelian Group	Spectrum
Commutative Ring	E_∞- Ring
Ring of Integers $\mathbb{Z}$	Sphere Spectrum S

Table 1: Analogies[5]

To every E_∞- ring R, we can associate an ∞- category $\text{Mod}_R(\text{Sp})$, of R- *modules spectra*, that is to say, modules over R, in the ∞- category of spectra. If M, and N, are R- modules spectra, we will denote the space $\text{Hom}_{\text{Mod}_R(\text{Sp})}(M, N)$, as their applications space.

Motivated for the analogies given in the table 1, we can give the following definition:

Def. 3. 1. A derived moduli problem is a functor X, from the ∞- category CAlg(Sp), of E_∞- rings to the ∞- category $\mathscr{C}$, of spaces.

4 Derived Moduli Problems and their Dualities: Main Results

Let $A, B \in \text{Alg}_{aug}^{(n)}$. Let $X \in \text{Moduli}_n$, the formal E_n- moduli problems on k. Let $\Phi_{\mathcal{R}}\text{F}$, $\text{G}\Psi_{\mathcal{R}}, \forall$ R, the functors defined and determined by the *theorem 6. 1*, $\forall$ R , a k- module that is an A- module. Then the isomorphism that represent these functors, are translated in the equivalences of ∞- categories:

$$\text{Moduli}_n \underset{\text{G}\Psi_{\mathcal{R}}}{\overset{\Phi_{\mathcal{R}}\text{F}}{\leftrightarrows}} \text{Alg}_{aug}^{(n)}, \tag{4.1}$$

which is true by the *theorem 6. 1.*[5-7]

We consider the following definition to establish the existence of the adjunct left and right functors F, G, that appear by integral transforms that are involved in the k- modules level [6].

Def. 4. 1. Let $X \in \text{Moduli}_n$, be formal E_n- moduli problems over k, and let A, be an augmented E_n- algebra over A. We will say that a natural transformation

$$\alpha : X \to \Psi(A), \tag{4.2}$$

reflects X, if, for every augmented E_n- algebra B, over k, composition with α, induces a homotopy equivalence:

$$\text{Hom}_{\text{Alg}^{(n)}_{\text{aug}}}(A, B) \to \text{Hom}_{\text{Moduli}_n}(X, \Psi(B)), \tag{4.3}$$

We let Moduli^0_n, denote the full sub-category of Moduli_n, spanned by these formal E_n- moduli problems X, for which there exists a map (4. 2) which reflects F. In this case, the map (4. 2) is well-defined up to canonical equivalence, considering in particular that we can regard the construction $X \mapsto A$, as defining a functor

$$\Phi : \text{Moduli}^0_n \to \text{Alg}^n_{\text{aug}}, \tag{4.4}$$

The functor Φ, is left adjunt to Ψ, in the sense that for every $X \in \text{Moduli}^0_n$, and every $B \in \text{Alg}^{(n)}_{\text{aug}}$, we have a canonical homotopy equivalence

$$\text{Hom}_{\text{Moduli}_n}(X, \Psi(B)) \to \text{Hom}_{Alg^{(n)}_{\text{aug}}}(\Phi(X), B), \tag{4.5}$$

Indeed, since the functor $\Psi : \text{Alg}^{(n)}_{\text{aug}} \to \text{Moduli}_n$, preserves small limits, one can deduce the existence of a left adjoint to Ψ, using the adjoint functor theorem. In other words, it follows formally that $\text{Moduli}^0_n = \text{Moduli}_n$. However, we will establish this equality by a more direct argument, which will help us to compute with the functor Φ .

Also we consider the following property due to Koszul self-duality of the little n- cubes operad [7, 8]:

$$\text{Alg}^{(n)}_{\text{aug}} \overset{\Phi^{-1}}{\cong} \text{Moduli}_n \subseteq \text{Fun}(\text{Alg}^{(n)}_{\text{sm}}, \mathscr{S}) \tag{4.6}$$

Theorem (F. Bulnes, I. Verkelov) 4. 1. Considering the functors Φ, Ψ, with the before properties (4. 1), (4. 3), (4. 5) and (4. 6), we have the following scheme

$$\mathrm{Hom}_{\mathrm{Moduli}_n}(X, \mathrm{Spec}(B)) \cong \mathrm{Hom}_{\mathrm{CAlg(Sp)}}(B, \mathscr{S}), \tag{4.7}$$

Proof. The demonstration is very immediate using the mentioned properties inside the hypothesis of the Theorem. However is necessary establish some fine details on the acting of the functors F, G, that appear in the *Koszul duality application*[8] and the relative details on the inverse limits to obtain Spf, [1] in the context of "CRings", CAlg(Sp).

Indeed, if $A \cong \varprojlim_\alpha A_\alpha$, is a pro-object of $\mathrm{Alg}_{\mathrm{sm}}^{(n)}$, then

$$\mathrm{Spf}A \cong \varinjlim_{\alpha} \mathrm{Spec}A, \tag{4.8}$$

then $\mathrm{Spf}A \in \mathrm{Moduli}_n$.

For other side, considering A, a E_n- small algebra over k, and $\mathrm{Spec}A \in \mathrm{Moduli}_n$, denoting the representation functor $\Phi^{-1} : \mathrm{Alg}_{\mathrm{sm}}^{(n)} \to \mathscr{S}$, given by the formula

$$(\mathrm{Spec}A)(B) = \mathrm{Hom}_{\mathrm{Alg}_{\mathrm{sm}}^{(n)}}(A, B) = \Psi(D\,A)(B)$$
$$= G\Psi(B) = G\Psi_{\mathcal{R}} \in \mathrm{Spec}B \in \mathrm{Fun}(\mathrm{Alg}_{\mathrm{sm}}^{(n)}, \mathscr{S}),$$

To the functor F, the existence of an arbitrary element $\mathrm{Spec}B$, that fall inside of $\mathrm{Alg}_{\mathrm{aug}}^{(n)}$, needs the additional arguments as $\mathrm{Moduli}_n^0 = \mathrm{Moduli}_n$, which was mentioned before. Then the compute of the functor Ψ , can be realized easily .

[1]If $A \cong \varprojlim_\alpha A_\alpha$, is a pro-object of $\mathrm{Alg}_{\mathrm{sm}}^{(n)}$, we let the functor

$$\mathrm{Spf}(A) : \mathrm{Alg}_{\mathrm{sm}}^{(n)} \to \mathscr{S},$$

as the functor given by the formula

$$B \mapsto \mathrm{Hom}_{\mathrm{Pro}(\mathrm{Alg}_{\mathrm{sm}}^{(n)})}(A, B) \cong \varinjlim_\alpha \mathrm{Hom}_{\mathrm{Alg}_{\mathrm{sm}}^{(n)}}(A_\alpha, B).$$

Then by (4. 1) all equivalences are satisfied in the context of the *moduli schemes* and "CRings". Due to that the functor $\Phi : \text{Moduli}_n \rightarrow \text{Alg}^{(n)}_{\text{aug}}$, is faithful then is followed all the scheme (4. 7).

As was mentioned before, the left and right functors F,G, appear by the integral transforms that are involved in the k- modules level. If we consider that these k- modules are $D_{G/H}$- modules then the equivalences given by the Penrose transform [9]

$$H^0(X, \mathcal{L}_\lambda) \cong \text{Ker}(\tilde{U}, \mathcal{Q}_{BRST}), \tag{4.9}$$

are translated in the equivalences [9, 10]:

$$\text{M}(D_{G/H} - \text{modules } \ G - \text{equivariants}) \underset{\overrightarrow{\ G\Psi_{\mathcal{R}}\ }}{\overset{\Phi_{\mathcal{R}}\text{F}}{\longleftarrow}} \text{M}_G(\mathfrak{g}, H), \tag{4.10}$$

which are translated in the isomorphism the Hecke categories [9]:

$$\mathcal{H}_{G^\wedge} \leftrightarrows \text{M}(\tilde{\mathfrak{g}}, Y), \tag{4.11}$$

where the Lie algebra $\tilde{\mathfrak{g}}$, is the loop extension of the loop algebra $\mathfrak{g}(t)$.

We consider the role of $\mathscr{C}$, in the theory of ∞- categories as the analogous to the ordinary category of sets in classical category theory and the Yoneda embedding defined by (3. 1) and (3. 2) for any ∞- category $\mathscr{C}$. Then in particular to a graded algebra $H^\bullet(Bun_G, \mathcal{D}^s)$, obtained from a Yoneda embedding, and generated by one copy of $\text{H}^\vee$, over $H^0 \cong \mathbb{C}[\text{Op}_{L_G}]$, is had that on a disk [2]:

Theorem (E. Frenkel, C. Teleman) 4. 2. The Yoneda algebra $\text{Ext}_{\mathcal{D}^s(\text{Bun}_G)}(\mathcal{D}^s, \mathcal{D}^s)$, is abstractly A_∞- isomorphic to (the strictly skew-commutative one) $\text{Ext}^\bullet_{\text{Loc}^L_G}(\mathcal{O}_{\text{Op}^L_G}, \mathcal{O}_{\text{Op}^L_G})$.

Considering a full subcategory of sheaves in $\mathscr{C} = \mathrm{Coh}(\mathrm{Loc}_{L_G})$, then we have:

$$H^{\bullet}(\mathfrak{g}[[z]], \mathfrak{g}; V_{\mathrm{crit}}) \cong \Omega^{\bullet}[\mathrm{Op}_{L_G}(D)], \tag{4.12}$$

Then considering an A_{∞}- enhancement of (4.12), that is to say, an algebra in $\mathrm{Alg}^{(n)}_{\mathrm{aug}}$, then we can give the isomorphism

$$H^{0}(\mathfrak{g}[[z]]; V_{\mathrm{crit}}) \cong \mathrm{Ker}(\mathrm{Bun}^{\circ}G, \partial), \tag{4.13}$$

which is the Penrose transform to a functor $\mathrm{Fun}(\mathcal{D}^{\mathrm{top}}, \mathscr{S})$. Here $\mathrm{Bun}^{\circ}_{G} = X$, where X, is the flag variety as the "quantum" version of the construction of an algebra symT.

Theorem (F. Bulnes) 4. 3. The integral transform of a functor in $\mathrm{Fun}(\mathcal{D}^{\mathrm{top}}, \mathscr{S})$, is the functor Spec, of an algebra symT.

Proof. We consider the scheme given in (4. 7) with the considerations $\mathscr{S} = (\mathrm{Calg}^{\mathrm{aug}}_{k})^{\mathrm{opp}}$, and $B = \mathrm{SymT} \subset \mathrm{Calg}^{\mathrm{aug}}_{k}$. Then an element in $\mathrm{Fun}(\mathcal{D}^{\mathrm{top}}, \mathscr{S})$, is obtained applying an integral transform $\Phi_R F$, defined for moduli spaces equivalence (4. 10) where is formally followed that $\mathrm{Moduli}^{0}_{n} = \mathrm{Moduli}_{n}$. But ,this particular equality can be translated to an isomorphism between categories where the functor $\Phi_R F$, image is a moduli stack given for a manifold Y, which is the spectrum of some ∞- algebra Sym, of $\mathrm{CAlg}^{\mathrm{aug}}_{k}$. But this is true to a hypercohomology where their dimension is only 1 or 0, to the quantum version of the corresponding cohomology space isomorphic to this hypercohomology [3]. Finally $\Phi_R F(\mathrm{SymT}) = \mathrm{Spec}_{X}(\mathrm{Sym}T\mathrm{X}) = \mathrm{T}^{\vee}\mathrm{X} = Y$, to $\mathrm{Bun}^{0}_{G} = \mathrm{X}$, being X, moduli stack that is a thick flag manifold. Filtering X, stay us with the elements of X, of the cohomology space $H^{q}(\mathrm{Bun}_{G}, \mathrm{Sym}T)$, where $q = 0, 1$, only. ◆

Remarks. The before consider an arbitrary base stack with affine diagonal (not necessarily perfect). Then to describe integral transforms relative to such a base, we will utilize the simple behavior of ∞- categories of sheaves under affine base change. Likewise corresponding modules of sheaves satisfy in particular to D-modules:

Derived Moduli Problems	Moduli Stacks
$X:\mathcal{R}\to \mathrm{Gpd}$	Pre-stacks
$\mathcal{D}:(\mathrm{CAlg}_k^{\mathrm{aug}})^{\mathrm{top}}\to \mathrm{Lie}_k$	$T^\vee \mathrm{Bun}_{L_G}, T^\vee \mathrm{Bun}_G$ (Higgs bundles)
$X:\mathrm{Vect}_k^{\mathrm{dg}}\to \mathrm{CAlg}_k$	Pre-Stacks on CAlg_k
$X:\mathrm{CAlg}_k^{\mathrm{sm}}\to \mathscr{C}$	$\mathrm{SpecSym}T(\mathrm{Bun}_G), \mathrm{SpecSym}T(\mathrm{Bun}_{L_G})=Y$
	Calabi-Yau Manifold
$C:\mathrm{Lie}_k\to (\mathrm{CAlg}_k^{\mathrm{aug}})^{\mathrm{opp}}$	$\mathrm{SymBun}_{L_G}, \mathrm{SymBun}_G$

Table 2: Derived Moduli Problems and their Moduli Stacks

(a) $\mathcal{R}$, is a category of commutative rings. In some special cases, the stackification can be described in terms of torsors for affine group schemes or the generalizations.

(b) Bun_{L_G}, is a ${}^LG$- bundle. The space $T^\vee \mathrm{Bun}_{L_G}$, is their corresponding cotangent bundle.

5 Applications: Integrals to the field equation d(da) $= 0$, [3].

We consider the integrals of the cohomological class $H^0(\mathfrak{g}[[z]]; V_{\mathrm{crit}}) \cong \mathrm{Ker}(\mathrm{Bun}^\circ G, \partial)$, which represent a solution to the field equation $Isomd\mathrm{h} = 0$. Their integrals are those whose functors image will be in $\mathrm{Spec}_H\mathrm{SymT}(OP_{l_G}(D))$, where $OP_{L_G}(D)$, is the variety of opers on the formal disk D, or neighborhood of all point in a surface Σ, in a complex Riemannian manifold $\mathbb{M}$. Its used to exhibit a short classification of cocycles of coherent D- modules and their re-interpretation in field theory as D- branes. The corresponding moduli stack are Higgs bundles.

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